# **Tutorial for Section 1.2**

# Fundamentals of the Analysis of Algorithm Efficiency

**Exercise 1**

For each of the following algorithms, indicate: (i) a natural size metric for its inputs. (ii) its basic operation (iii) whether the basic operation count can be different for inputs of the same size:

1. Computing the average of *n* numbers
2. Computing
3. Finding the smallest element in a list of *n* numbers
4. Reverse display a list of *n* numbers
5. Reverse a list of *n* numbers

**Answer**

1. (i) n, (ii) addition, (iii) no.
2. (i) the magnitude of n, i.e., the number of bits in its binary representation, (ii) multiplication, (iii) no.
3. (i) n, (ii) comparison of two numbers, (iii) no (for the standard list scanning algorithm).
4. (i) n (ii) display of numbers, (iii) no.
5. (i) n (ii) swap, (iii) no.

**Exercise 2**

Consider the definition-based algorithm for finding the difference between two matrices. What is its basic operation? How many times is it performed as a function of the matrix order *n*? As a function of the total number of elements in the input matrices?

**Answer**

The basic operation is the subtraction of two numbers. It is performed n2 times (once for each of n2 elements in the matrix being computed). Since the total number of elements in two given matrices is N = 2n2 the total number of subtractions can also be expressed as n2 = N/2.

**Exercise 3**

Prove that the number of bits in the binary representation of a positive integer *n* is .

**Hint 1**: Consider the smallest and largest integers *n* having *b* bits.

**Hint 2**:

**Answer**

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Description automatically generated with medium confidence

A close-up of a mathematical equation

Description automatically generated

**Exercise 4**

Gaussian elimination, the classic algorithm for solving systems of *n* linear equations in *n* unknowns, requires about multiplications, which is the algorithm’s basic operation.

1. How much longer should you expect Gaussian elimination to work on a system of 1000 equations versus a system of 500 equations?
2. You are considering buying a computer that is 1000 times faster than the one you currently have. By what factor will the faster computer increase the sizes of systems solvable in the same amount of time as on the old computer?

**Answer**

1. A system of 1000 equations has a double size compared to a system of 500 equations.

So, the answer is 8 times longer.

So,

The answer is 10

**Exercise 5**

For each of the following functions, indicate how much the function’s value will change if its argument is increased threefold.

1. *n*
2. *n*3
3. *n*2
4. *n*!
5. 2*n*

**Hint**: Use either the difference between or the ratio of and .

**Answer**



**Exercise 6**

***Invention of chess***

1. According to a well-known legend, the game of chess was invented many centuries ago in northwestern India by a certain sage. When he took his invention to his king, the king liked the game so much that he offered the inventor any reward he wanted. The inventor asked for some grain to be obtained as follows: just a single grain of wheat was to be placed on the first square of the chess board, two on the second, four on the third, eight on the fourth, and so on, until all 64 squares had been filled. If it took just 1 second to count each grain, how long would it take to count all the grain due to him?

**Hint**: Use the formula

1. How long would it take if instead of doubling the number of grains for each square of the chessboard, the inventor asked for adding two grains?

**Answer**

1. The total number of grains due to the inventor is:

If it took just one second to count each grain, the total amount of time needed to count all these grains comes to about 585 billion years!!

1. The total number of grains is:

With the same speed of counting one grain per second, he would have needed less than one hour and fourteen minutes to count his modest reward.